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This report outlines an initial set of approaches for calculating the far field radiation and diffraction waves generated by a ship advancing at constant speed and oscillating in the presence of ambient ocean waves. The commonly used strip theory for obtaining the near field solution is described in some detail. Several approaches are given for obtaining the far field singularity distribution. These include conversion of the calculated near field singularity distribution or forces to a line of singularities, or use of an amplitude function which represents an integrated effect of the distribution. It is pointed out that the far field asymptotic behavior of the waves due to stationary

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20. ABSTRACT (Continued)

singularities is well known and relatively simple to calculate. Several simplified approaches for calculating the corresponding behavior for the considerably more complex forward speed case are briefly discussed.

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NRL Memorandum Report 5028

Calculation of Far Field, Ship Radiation and Diffraction Waves

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March 28, 1983

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CALCULATION OF FAR FIELD, SHIP RADIATION AND DIFFRACTION WAVES

1. INTRODUCTION

This memorandum report outlines the initial set of proposed approaches to obtain the far field radiation and diffraction waves generated by a ship advancing at constant speed and oscillating in the presence of ambient ocean waves. Radiation waves are caused by the oscillation of the ship while the diffraction waves refer to the disturbance to the incoming ambient waves caused by the presence of the ship.

The report first presents the linearized mathematical formulation which is often used to solve these problems. The assumptions underlying this formulation are briefly stated. The further simplifications which are required to arrive at the commonly used strip theory approach, as implemented in the DTNSRDC Ship Motion Program, are described. It is pointed out that this is a two-dimensional near field solution which is accurate near the ship hull. The mathematical convenience as well as the limitations resulting from these simplifications are discussed. The principal differences and similarities of the radiation and diffraction problems in the strip theory approach are indicated.

The principal interest in the large majority of previous studies has been the calculation of the near field flow in order to obtain the added mass, damping, and exciting forces on the ship. The two steps required to convert the near field solutions to the calculation of the far field wave pattern are outlined in some detail. The first step is to convert the near field solution to an equivalent distribution of far field singularities placed on or near the hull of the ship. The second step requires the asymptotic evaluation of the far field wave pattern due to these singularities.

Three approaches are described for obtaining the far field singularity distribution. The first approach involves directly converting the calculated near field two-dimensional singularity distribution to an equivalent distribution of three-dimensional far field singularities by using the method of matched asymptotic expansions. The second approach is somewhat similar to the first. It involves first converting the calculated or measured forces on the two-dimensional strips to a distribution of near field singularities and then using the first approach to obtain the far field singularity distribution. This approach makes use of the abundant data on the added mass, damping, and exciting forces acting on ship hulls. The third approach characterizes the entire singularity distribution on the ship hull by an amplitude function which represents an integrated effect of the distribution.

It is pointed out that asymptotic expressions have been derived for the far field behavior of pulsating sources and dipoles which are otherwise stationary. Thus, the evaluation of the far field wave pattern for ships with zero speed is relatively simple. The situation is quite different for the case of forward speed, for which the formulas and wave pattern of the singularities are considerably more complex. There have been relatively few studies which consider the far field behavior of these singularities. These studies are concerned with obtaining some general features of the far field behavior and are largely written in French and German. In view of the complexity of the forward speed case, some alternate simpler approaches (which may be suitable for low and moderate speed cases) are briefly indicated.

Manuscript approved December 17, 1982.

2. LINEARIZED FORMULATION OF PROBLEM

The problem is most often formulated for a coordinate system (x, y, z), shown in Fig. 1, moving in the x-direction with the mean forward speed U of the ship. The moving coordinate system (x, y, z) is related to the fixed spatial coordinate system (x_0, y_0, z_0) by

$$(x, y, z) = (x_0 - Ut, y_0, z_0) \tag{1}$$

where t is time. Also, the frequency of encounter ω in the moving coordinate system is related to the frequency ω_0 of incident waves in the fixed coordinate system by

$$\omega = \omega_0 - Uk_0 \cos\beta = \omega_0 - \frac{U\omega_0^2}{g}\cos\beta \tag{2}$$

where β is the angle between the direction of propagation of the incident wave and the x-direction

$k_0 = \omega_0^2/g = 2\pi/\lambda$ is the wavenumber

λ is the wavelength.

The assumptions are made that the fluid is inviscid and incompressible, the flow is irrotational, and surface tension effects may be neglected. Under these assumptions, the flow field is given by the velocity potential $\Phi\ (x, y, z)$ which satisfies Laplace's equation

$$\nabla^2\Phi(x, y, z) = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0. \tag{3}$$

The total potential Φ may be written as the sum of the steady potential $U\bar{\phi}$, which gives rise to the Kelvin waves, and the unsteady potential ϕ

$$\Phi = U\bar{\phi} + \phi e^{i\omega t}. \tag{4}$$

The principal interest in the present report is the unsteady potential ϕ. Assuming that the amplitudes of the incident, radiation, diffraction, and Kelvin waves are all small leads to the linearized formulation given below. Nonlinear formulations are given by Ogilvie and Tuck,[1] Newman,[2] and Maruo.[3] However, it should be noted that even the linearized problem is of considerable difficulty, and most of the current investigations focus on solving various simplifications of the linearized formulation.

Under the above linearizing assumptions, the potential ϕ may be further decomposed into separate potentials due to the incident wave, each of the six rigid body oscillations shown in Fig. 1, and the diffracted wave. Assuming the motions to be sinusoidal at the encounter frequency ω, the notation shown in Fig. 1 suggests that the motions of the ship are given by

$$(\xi_1, \xi_2, \xi_3, \Omega_1, \Omega_2, \Omega_3)e^{i\omega t} \equiv (\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)e^{i\omega t}. \tag{5}$$

The unsteady potential ϕ may then be written as

$$\phi = [A(\phi_0 + \phi_7) + \sum_{j=1}^{6} \xi_j\phi_j]e^{i\omega t} \tag{6a}$$

where A is the amplitude of the incident wave,

$$\phi_0 = \frac{ig}{\omega_0}\exp[k_0(z - ix\cos\beta - iy\sin\beta)], \tag{6b}$$

is the potential for an incident wave of unit amplitude whose direction of propagation is at angle β with the x-axis, and ϕ_7 is the diffraction potential. The original problem is thus restated as the separate solution of the diffraction problem where incident waves act upon the equilibrium submerged hull surface, and the radiation problem where the ship undergoes prescribed oscillations in the absence of incident waves.

On the mean free surface $z = 0$ each of the above potentials ϕ_j, $j = 0$ to 7, must satisfy the linearized free surface condition

$$\frac{d^2\phi_j}{dt^2}(x_0 - Ut,\ y,\ z) + g\frac{\partial\phi_j}{\partial z} =$$

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)^2 \phi_j + g\frac{\partial\phi_j}{\partial z} =$$

$$\frac{\partial^2\phi_j}{\partial t^2} - 2U\ \frac{\partial\phi_j}{\partial t}\ \frac{\partial\phi_j}{\partial x} + U^2\frac{\partial^2\phi_j}{\partial x^2} + g\frac{\partial\phi_j}{\partial z} = 0$$

on $z = 0$, $j = 0, 1, \ldots, 6, 7$. (7)

On the mean submerged hull surface S, the diffraction potential ϕ_7 must satisfy the kinematic condition

$$\frac{\partial\phi_7}{\partial n} = -\ \frac{\partial\phi_0}{\partial n} \text{ on } S \tag{8}$$

where n is the normal to the hull surface. The kinematic condition on S to be satisfied by the radiation potentials for the case of a ship moving at forward speed U have been derived by Timman and Newman[4]

$$\frac{\partial(\sum_{j=1}^{6} \xi_j\phi_j)}{\partial n} = \{i\omega\vec{\alpha} + \nabla\times\ [\vec{\alpha} \times \nabla(U\vec{\phi})]\} \cdot \vec{n} \tag{9}$$

where $\vec{\alpha}$ is the displacement of a point on S. Recalling that the disturbance associated with the steady forward motion is small, and expressing $\vec{\alpha}$ in terms of the oscillatory translations $\vec{\xi}$ and rotations $\vec{\Omega}$ defined in Eq. (5) and Fig. 1, the above equation becomes[2,5]

$$\frac{\partial\phi_j}{\partial n} = i\omega n_j + Um_j \text{ on } S,\ j = 1,\ \ldots,\ 6 \tag{10}$$

where

$$(n_1,\ n_2,\ n_3) = \vec{n} \tag{11}$$

$$(n_4,\ n_5,\ n_6) = (\vec{r} \times \vec{n}) \tag{12}$$

$\vec{r}$ is the radius vector from the origin to a point on the hull surface

$$(m_1,\ m_2,\ m_3,\ m_4) = 0$$

$$m_5 = n_3,\ m_6 = -\ n_2. \tag{13}$$

The solution of even the above linearized problem is a formidable task. For example, the computer cost for the method developed by Chang[6] to solve the above problem is nearly two orders of magnitude larger[7] than the corresponding cost for the widely used DTNSRDC Ship Motion Program (SMP)[5,8], which uses the simpler strip theory approach. This theory, which is described in greater detail later in this report, essentially converts the complex three-dimensional problem to a series of simpler two-dimensional problems.

3. SIMPLIFICATIONS TO LINEARIZED THEORY

Most of the simplifications to the above linearized theory are based on making one or more of the following assumptions regarding the magnitude of the ship and wave parameters[2,3,9]

$$B/L << 1 \tag{14a}$$

$$T/L << 1 \tag{14b}$$

$$\omega^2 L/g = 2\pi L/\lambda << 1 \tag{14c}$$

$$\omega^2 L/g = 2\pi L/\lambda >> 1 \tag{14d}$$

where B is the beam (width) of the ship
L is the length of the ship
T is the draft of the ship.

Assumptions (14a) and (14b) lead respectively to thin ship and flat ship (or planing ship) theories. Assumptions (14c) and (14d) lead respectively to low frequency (or large wavelength) and high frequency (or small wavelength) theories. A relatively large number of studies are based on slender body theory, in which assumptions (14a) and (14b) are concurrently used. A particular form of slender body theory, in which assumption (14d) is also used, leads to the widely used strip theory which is described in detail in the following section.

For motions in the vertical plane (surge ξ_1, heave ξ_3, and pitch ξ_5), thin ship theory leads to a simple, direct solution for the potential ϕ, as follows. By using Green's Theorem, ϕ can generally be expressed as the following integral over the hull surface S

$$\phi = \frac{1}{2\pi} \iint_S \left[G_S \frac{\partial \phi}{\partial n} - \phi \frac{\partial G_S}{\partial n} \right] dS \tag{15}$$

where G_S is a pulsating source which satisfies Laplace's Eq. (3) and the free surface condition (7). The use of thin ship theory allows the integral to be evaluated over the vertical ($y = 0$) centerplane of the ship. The derivative $\partial/\partial n$ may then be approximated by $\partial/\partial y$. The crucial simplification results from the fact the that $\partial G_S/\partial n = \partial G_S/\partial y = 0$ on the centerplane $y = 0$ since G_S is an even function in y. Equation (15) then simplifies to

$$\phi = \frac{1}{2\pi} \iint_S G_S \frac{\partial \phi}{\partial y} dS \tag{16}$$

where $\partial\phi/\partial y$ is the prescribed velocity on the ship hull and is given in Eqs. (8) to (13). The above equation then represents a direct solution for ϕ.

For the case of a nonoscillating ship advancing at constant speed U, thin ship theory yields useful results for the steady-state flow field. For the case of a ship executing oscillations in the vertical plane, Peters and Stoker[10] obtain the disappointing result that the oscillations cause no disturbance to the fluid, i.e., the added mass and damping forces are zero. Mathematically, the difference between the steady-state and oscillating cases may be viewed as follows. The thin ship assumption, given by Eq. (14a), may be rewritten as

$$B/L = 0\ (\epsilon_B),\ \epsilon_B << 1. \tag{17}$$

For a ship advancing at finite velocity, the following dimensionless form for U may be taken to be order 1

$$F^2 = U^2/gL = 0(1) \tag{18}$$

where F is the Froude number. The linearizing assumptions stated under Eq. (4) imply that the ship oscillations ξ_j are small

$$\xi_j = 0\ (\epsilon_\xi),\ \epsilon_\xi << 1,\ j = 1, \ldots, 6. \tag{19}$$

Thus, the flow disturbance caused by the steady velocity is of first order, while the disturbance caused by the ship oscillations is of higher order. Physically, the results of Peters and Stoker simply state that a thin vertical disk excuting small oscillations in the vertical plane creates disturbances which are negligible to first order. Newman[11] has carried out the analysis to second order and obtained nonzero expressions for the added mass and damping forces. However, to quote Newman, "Unfortunately, the second-order equations are rather complex...as might be expected from the use of a systematic perturbation procedure." Accordingly, his proposed approach has not gained acceptance as a practical computational method.

It should be noted that thin ship theory predicts nonzero hydrodynamic disturbances for ship oscillations in the lateral y-direction (sway ξ_2, roll ξ_4, and yaw ξ_6). However, the crucial direct solution feature for motions in the vertical plane is lost in this case. For motions in the lateral direction, the fundamental singularity is the dipole G_L given by

$$G_L = \frac{\partial G_S}{\partial y}. \tag{20}$$

The quantity $\partial G_L/\partial n = \partial G_L/\partial y = \partial^2 G_S/\partial y^2$ is an even function in y and is not necessarily equal to zero on the vertical centerplane $y = 0$. Thus both terms on the right hand side of Eq. (15) must be retained.

The lack of success of first order thin ship theory led Peters and Stoker[10] to consider flat ship theory, which uses assumption (14b). Newman[11,12] points that this theory leads to the considerable complexity of solving singular integral equations.

The low frequency assumption (14c) leads to two cases of general interest depending on the magnitude of the Froude number F defined in Eq. (18):

$$F^2 = 0(1) \tag{21a}$$

$$F^2 \rightarrow 0. \tag{21b}$$

Assumptions (14c) and (21a) reduce the free surface condition given in Eq. (7) to

$$U^2 \frac{\partial^2 \phi_j}{\partial x^2} + g \frac{\partial \phi_j}{\partial z} = 0 \text{ on } z = 0 \tag{22}$$

and the hull boundary condition (10) to

$$\frac{\partial \phi_j}{\partial n} = U\ m_j \text{ on } S,\ j = 1, \ldots, 6. \tag{23}$$

Thus, all dependence on ω is removed, resulting in a purely steady-state problem for a nonoscillating ship. Assumptions (14c) and (21b) reduce the free surface condition to

$$\frac{\partial \phi_j}{\partial z} = 0 \tag{24}$$

and the hull boundary condition to

$$\frac{\partial \phi_j}{\partial n} = i\omega nj \text{ on } S,\ j = 1, \ldots, 6. \tag{25}$$

Equation (25) shows that the oscillatory character of the ship motions has been preserved. This large wavelength, low speed theory leads to considerable simplifications. For example, Newman[13] shows that the diffraction potential ϕ_7 can be approximated by

$$\phi_7 \approx \frac{i}{\omega} \left(\frac{\partial \phi_0}{\partial x} \phi_1 + \frac{\partial \phi_0}{\partial y} \phi_2 + \frac{\partial \phi_0}{\partial z} \phi_3 \right). \tag{26}$$

One simplification in the derivation of the above equation is that the derivatives of ϕ_0 can be taken to be constants, evaluated at a convenient location around the body. The present author[14] has exploited the fact that this theory leads to body translational motions which tend to follow those of the surface wave to derive expressions for the forces acting on wide classes of buoys. However, this case is not of primary interest for the present study due to the fact that Eq. (24) shows that the free surface behaves like a flat plane. This implies the absence of radiation waves (and the damping forces which give rise to these waves).

4. STRIP THEORY (NEAR FIELD SOLUTION)

Strip theory is a particular form of slender body theory which seeks to reduce the three-dimensional problem defined by Eqs. (1) to (13) to a series of simpler two-dimensional problems for cross sections (or strips) at various stations along the longitudinal x-axis. The resultant theory gives a near field solution applicable only in the neighborhood of the hull. The extension to the far field will be discussed in the following section. The near field solution is appropriate for computing the wave pattern near the ship and the forces acting on the hull. A number of different versions of strip theory and slender body theory have been used to analyze radiation and/or diffraction problems; see, for example, the survey by Newman.[9] The description given below of the strip theory for the radiation problem follows closely the formulation in the widely used DTNSRDC Ship Motion Program (SMP).[5,8] The description of the solution for the diffraction potential differs from the formulation given in SMP. In SMP, the diffraction potential is not computed explicitly. Instead, the diffraction forces are computed from the radiation potentials.

4.1 Radiation Problem

In the SMP formulation for the radiation problem, the slender body assumptions, Eqs. (14a) and (14b), are rewritten in the following form involving the derivatives and normals in the neighborhood of the hull

$$\frac{\partial}{\partial x} = 0(1),\ \frac{\partial}{\partial y} = 0(1/\epsilon),\ \frac{\partial}{\partial z} = 0(1/\epsilon)$$
$$n_x = 0(1),\ n_y = 0(1/\epsilon),\ n_z = 0(1/\epsilon). \tag{27}$$

These assumptions reduce the three-dimensional Laplace's Eq. (3) to the following two-dimensional form in the y-z plane

$$\frac{\partial^2\phi_j}{\partial y^2} + \frac{\partial^2\phi_j}{\partial z^2} = 0,\ j = 1,\ \ldots,\ 6. \tag{28}$$

In order to reduce the free surface condition, Eq. (7), to two-dimensional form near the hull, assumption (14c) is used in the form

$$\frac{\partial}{\partial t} >> U\frac{\partial}{\partial x}. \tag{29}$$

This reduces Eq. (7) to the following standard speed independent two-dimensional form

$$-\omega^2\phi_j + g\frac{\partial\phi_j}{\partial z} = 0,\ j = 1,\ \ldots,\ 6. \tag{30}$$

The speed dependent boundary condition in Eq. (10) is eliminated as follows. The potentials ϕ_j are divided into two parts,

$$\phi_j = \phi_j^0 + \frac{U}{i\omega}\phi_j^U,\ j = 1,\ \ldots,\ 6 \tag{31}$$

where the superscripts 0 and U respectively denote speed independent and speed dependent parts, resulting in the revised boundary conditions

$$\partial\phi_j^0/\partial n = i\omega n_j,\ j = 1,\ \ldots,\ 6 \tag{32a}$$
$$\partial\phi_j^U/\partial n = i\omega m_j,\ j = 1,\ \ldots,\ 6. \tag{32b}$$

Use of the relations given in Eq. (13) leads to the following expression for ϕ_j, completely in terms of the speed independent potentials ϕ_j^0

$$\phi_j = \phi_j^0, \quad j = 1, 2, 3, 4$$

$$\phi_5 = \phi_5^0 + \frac{U}{i\omega} \phi_3^0$$

$$\phi_6 = \phi_6^0 - \frac{U}{i\omega} \phi_2^0. \tag{33}$$

In addition, the assumptions of strip theory which imply that there is no interaction between cross sections lead to the following simple relations between the rotational motions pitch ($j = 5$) and yaw ($j = 6$) and the corresponding translational motions heave ($j = 3$) and sway ($j = 2$)

$$\phi_5^0 = -x\phi_3^0, \; \phi_6^0 = x\phi_2^0. \tag{34}$$

The formulation contained in Eqs. (27) to (34) reduces the original three-dimensional problem to the solution of the four fundamental two-dimensional speed independent potentials

$$\phi_j^0(y, z), \; j = 1, 2, 3, 4$$

at a series of cross sections along the hull. The Frank close-fit source distribution method is used to solve this formulation.[15] Briefly, the method models only the $+y$ half of a given cross section, which is assumed to be symmetric with respect to the vertical x - z plane, as shown in Fig. 2. The actual continuous half-cross section is approximated by a series of N straight line segments connecting $N + 1$ input points (y_i, z_i) where y_i is the half-breadth of the section at depth z_i. A pulsating line source of strength Q_i is placed over each line segment, where Q_i varies from segment to segment. The source strengths over the $-y$ half of the section are symmetric for the vertical modes ($j = 1, 3$) and antisymmetric for the lateral modes ($j = 2, 4$), as follows

$$Q_i(-y_i, z_i) \equiv Q_i(y_i, z_i) \quad j = 1, 3 \tag{35a}$$

$$Q_i(-y_i, z_i) \equiv -Q_i(y_i, z_i) \quad j = 2, 4. \tag{35b}$$

The kinematic boundary conditions given in Eq. (32a) are used to obtain a system of N complex algebraic equations for the complex strengths Q_i. This procedure is then repeated at a series of cross sections along the length of the ship.

The effect of forward speed is contained in the expressions for ϕ_5 and ϕ_6 given in Eq. (33) (which are directly related to ϕ_j^0) and in the expression for the radiation forces

$$F_{j2D} = \int_C p n_j \, dl$$

$$= \int_C -\rho (i\omega - U \frac{\partial}{\partial x}) \sum_{k=1}^{6} \xi_k \phi_k n_j \, dl, \; j = 1, \ldots, 6 \tag{36}$$

where F_{j2D} is the two-dimensional radiation force acting on a given cross section.
C is the perimeter of the cross section
ξ_k is the amplitude of the kth mode of motion.
The radiation forces F_j on the entire hull are then given by

$$F_j = \int_L F_{j2D}(x)\, dx, \quad j = 1, \ldots, 6. \tag{37}$$

In spite of the many simplifications listed above, SMP appears to give reasonably accurate results for forces and motions compared to experiment[5] and to a more complex three-dimensional approach.[6] Thus, it will be used in the present study as the initial approach for obtaining the near field solution. A more complex unified strip theory developed by Newman and Sclavounos,[16] which accounts for an interaction between ship cross sections, may be used at a later stage if the additional accuracy is required.

It should be noted that there are a number of inconsistencies in the above theory. Perhaps the most noticeable is the neglect of the convection term $U\partial/\partial x$ in the free surface condition, Eq. (30), and its inclusion in the force Eq. (36). Ogilvie and Tuck[1] have developed a more consistent strip theory which shows that most, but not all, of the higher order effects tend to cancel each other. The effect of the nonzero higher order corrections, which affect only the cross-coupling force coefficients, is not yet well estblished.

4.2 Diffraction Problem

SMP does not explicitly calculate the diffraction potential ϕ_7. Instead, the diffraction forces F_j^D are computed from the values of $\partial\phi_0/\partial n$ (where ϕ_0 is given in Eq. (6b)) and the previously calculated radiation potentials ϕ_j^0 and ϕ_j^U (Eqs. (31)-(34)) on the hull surface by using a generalized form of the Haskind relations, as follows

$$F_j^D = \rho \int\int_S \left[\phi_j^0 - \frac{U}{i\omega}\,\phi_j^U\right] \frac{\partial\phi_0}{\partial n}\, dS, \quad j = 1, \ldots, 6. \tag{38}$$

The Haskind relations were originally derived for the case of zero speed. However, Newman[17] generalized these relations to the forward speed case for rather general conditions. The principal case where the relations do not apply is for a thin ship in head seas, $\beta = 180$ deg.

The boundary condition (8) for ϕ_7 leads to several differences in the formulation from the previously discussed case for the radiation potentials ϕ_j. Equations (8) and (6b) show that

$$\frac{\partial\phi_7}{\partial n} = \frac{-\partial\phi_0}{\partial n} = \frac{-\partial}{\partial n}\,\frac{ig}{\omega_0}\,\exp\left[k_0\,(z - ix\cos\beta - iy\sin\beta)\right]$$

$$= f(k_0 = \omega_0^2/g,\ x,\ y,\ z,\ \beta). \tag{39}$$

On the other hand, the four fundamental radiation potentials ϕ_j^0, $j = 1, 2, 3, 4$ have the functional form

$$\phi_j^0 = f(\omega,\ y,\ z). \tag{40}$$

Since the x-dependence of ϕ_7 is periodic, it is usual to write ϕ_7 as the following product

$$\phi_7(x,\ y,\ z) = \Phi_7(y,\ z,\ \omega_0,\ \beta)\ e^{-il_0x} \tag{41}$$

where $l_0 = k_0 \cos\beta$

Φ_7 is a two-dimensional solution.

Substitution of the slender body assumptions (27), and Eq. (41) into the three-dimensional Laplace's Eq. (3) results in the following two-dimensional Helmholtz equation for $\Phi_7\ (y,\ z)$

$$\frac{\partial^2\Phi_7}{\partial y^2} + \frac{\partial^2\Phi_7}{\partial z^2} - l_0^2\,\Phi_7 = 0 \tag{42}$$

as opposed to a two-dimensional Laplace's equation for the radiation potentials. A second difference is that the substitution of Eq. (41) along with assumption (27) into the free surface equation (7) yields

$$-\omega_0^2\,\Phi_7 + g\,\frac{\partial\Phi_7}{\partial z} = 0 \tag{43}$$

whereas the free surface Eq. (30) for ϕ_j is in terms of the encounter frequency ω.

The need to solve the Helmholtz Eq. (42) instead of Laplace's Eq. (28) makes the diffraction problem somewhat more difficult since the singularities appropriate for the Helmholtz equation must be used. Faltinsen,[18] Troesch,[19] and Maruo[3] solve the above formulation for zero and low speed cases. Faltinsen finds that "It would be time consuming to evaluate the solutions....for a ship with arbitrary cross sections." and obtains results for only circular cross section cases.

Strictly speaking, the dependence of ϕ_7 on x shown in Eq. (41) violates the strip theory assumption that there is no interaction between cross sections. For this reason, strip theory approaches either bypass the calculation of ϕ_7, as in the previously described case of SMP which uses the Haskind relations to calculate the diffraction forces, or attempt to eliminate the x-dependence shown in Eq. (41) by setting $l_0 = 0$. This, in effect, simplifies the Helmholtz Eq. (42) to the simpler Laplace's Eq. (28). One obvious case where $l_0 = 0$ is for beam seas, $\beta = 90°$. Another case would appear to be for long waves such that

$$\left(k_0 = \frac{2\pi}{\lambda} = \frac{\omega_0^2}{g} \right) << 1. \tag{44}$$

However, this would violate the strip theory assumption (14d) that the waves are short. Newman[13] shows that strip theory is applicable to the diffraction problem over a restricted range of λ such that it is short compared to the ship length L but long compared to the beam B

$$B << \lambda << L. \tag{45}$$

In this restricted range, the diffraction potential is solved in a manner which is very similar to the radiation potentials[13]

$$\phi_7 = -i\omega_0 \, (\psi_2 - i \sin \beta \psi_3) \, e^{ik_0 x \cos \beta} \tag{46a}$$

$$\frac{\partial \psi_2}{\partial n}(y, z) = n_y, \quad \frac{\partial \psi_3}{\partial n}(y, z) = n_z \tag{46b}$$

where ψ_2 and ψ_3 are respectively the antisymmetric and symmetric two-dimensional potentials which satisfy Laplace's Eq. (28) and the free surface condition (43). For the case of $\beta = 45$ deg. Troesch[19] shows that the pressure distributions on the hull of an ore carrier obtained by using this simpler approach show only moderate differences from those obtained by using the more complex Helmholtz Eq. (42). Hence, the formulation outlined in Eqs. (46a) and (46b) will initially be used to compute ϕ_7.

The above shows that two-dimensional solutions for the diffraction potential are more difficult and/or are applicable over a more restricted range of parameters than the corresponding solutions for the radiation potentials. For fully three-dimensional approaches, such as that developed by Chang,[6] there is less difference between the radiation and diffraction problems since both are dependent on x.

5. FAR FIELD DISTRIBUTION OF SINGULARITIES

The near field singularity distribution obtained by the above strip theory must be converted to a singularity distribution which is appropriate for calculating the far field wave pattern. One obvious shortcoming of the near field solution is that the pulsating sources resulting from the two-dimensional formulation gives waves of constant amplitude at large distances from the hull. Physically, the waves attenuate to zero at large distances due to cylindrical spreading.

The present section will discuss three approaches for obtaining the equivalent far field singularity distribution. Since the formulation which will be initially used for the diffraction potential ϕ_7 is similar to the formulation for the radiation potentials ϕ_j, $j = 1, \ldots, 6$, the discussion essentially applies to both types of potentials.

Far from the ship, the details of the ship hull are lost, and the hull disturbance may be taken to be caused by a line of pulsating singularities placed on the x-axis along the length L of the ship. In general, the outer potential ϕ_j can be expressed in the form

$$\phi_j = \int_L \left[q_j(\xi) + d_j(\xi) \frac{\partial}{\partial_y} \right] G_{3D}(x - \xi, y, z) \, d\xi \tag{47}$$

where ξ is the location of the source along the x-axis
G_{3D} is a pulsating source translating at velocity U in the x-direction
$\partial G_{3D}/\partial y$ is a pulsating dipole
q_j is the strength of the source distribution
d_j is the strength of the dipole distribution.

The potential ϕ_j may be taken to be the corresponding outer solution for the inner radiation potentials ϕ_j, $j = 1, \ldots, 6$ as well as the inner diffraction potentials ψ_j, $j = 2, 3$. From symmetry considerations, $q_j = 0$ for the lateral modes ($j = 2, 4, 6$) and $d_j = 0$ for the vertical modes ($j = 1, 3, 5$).

5.1 Relation to Near Field Singularities

The most direct approach, which will be described first, is to relate the strengths of this far field line distribution to the strengths of the near field surface distribution calculated by using strip theory. Since the cross section shape is lost in the far field, the singularity distribution over each cross section may be condensed to a single resultant singularity strength as follows. For the symmetric vertical modes ($j = 1, 3, 5$), for which Eq. (35a) applies, the resultant source strengths at the cross section $x = \xi$, $\sigma_j(\xi)$, is given by the following summation

$$\sigma_j \equiv \sigma_j^0 + U\hat{\sigma}_j = 2 \sum_{i=1}^{N} Q_i \Delta s_i, \; j = 1, 3, 5 \tag{48}$$

where the first equality is suggested by Eq. (31)
N is the number of line segments modeling the $+y$ half of the cross section
Δs_i is the length of the ith line segment.

For the antisymmetric lateral modes ($j = 2, 4, 6$), for which Eq. (35b) holds, the resultant source strength σ_j is zero. However, the resultant dipole strength μ_j is nonzero

$$\mu_j \equiv \mu_j^0 + U\hat{\mu}_j = 2 \sum_{i=1}^{N} \frac{Q_i(y_i, z_i)}{2\, y_i} \Delta s_i, \; j = 2, 4, 6. \tag{49}$$

Newman,[2] and Ogilvie and Tuck[1] use the method of matched asymptotic expansions to derive expressions for the outer singularity strengths q_j and d_j in terms of the inner strengths σ_j and μ_j. The essence of the method is that the inner and outer solutions are matched in an overlap region r whose distance from the ship hull is large compared to the beam B or draft T and small compared to the length L

$$(B, T) << r << L. \tag{50}$$

To second order accuracy in ϵ, where $(B/L, T/L) = 0(\epsilon)$, the relation between the outer dipole strengths d_j and the inner strengths μ_j for the lateral modes is extremely simple

$$d_j = \mu_j = \mu_j^0 + U\hat{\mu}_j, \; j = 2, 4, 6. \tag{51}$$

That is, in slender body theory, there is negligible longitudinal interaction between cross sections for lateral motions and the strengths of the inner and outer dipoles coincide. (Recall, however, that the inner dipoles are two-dimensional while the outer dipoles are three-dimensional.) The relation for the vertical modes is somewhat more complex. In general, the outer source strengths q_j are related to the inner strengths σ_j by the following integral equation

$$q_j(x) - \frac{1}{2\pi i}\left(\frac{\sigma_j}{\bar{\sigma}_j} - 1\right) \int_L q_j(\xi) f(x - \xi) d\xi = \sigma_j(x), \; j = 1, 3, 5 \tag{52}$$

where $\bar{\sigma}_j$ is the complex conjugate of σ_j

$f(x - \xi)$ is an interaction function defined by Eq. (4.13) in Newman.[2] However, for the two limiting regions where

$$\lambda = 0(1) \tag{53a}$$

$$\lambda = 0(\epsilon) \tag{53b}$$

Eq. (52) may be approximated by

$$q_j \approx \sigma_j = \sigma_j^0 + U\hat{\sigma}_j, \ j = 1, 3, 5. \tag{54}$$

For these two wavelength regions, the inner and outer singularity strengths again coincide as in the case of the lateral modes.

In the present study, the initial approach will use Eqs. (51) and (54) to obtain the outer singularity strengths. If additional accuracy is required, the integral Eq. (52) will be used for the vertical modes.

5.2 Relation to Added Mass and Damping Forces

The second approach is somewhat more indirect than the first approach. In this approach, the computed forces acting on a cross section are first converted to a resultant inner singularity strength and then Eqs. (51) and (54) are used to obtain the outer singularity strengths. However, this approach does have the advantage that it would make use of the many previous calculations of the forces on various cross section shapes; see, for example, the extensive calculations by Porter[20] and the various results reported by Wehausen.[21]

One motivation for this approach is that in infinite fluid, there exist the following simple relations between the dipole strengths d_{ij} and the added mass coefficients A_{ij}[13]

$$2D: \ d_{ij} = \frac{-1}{2\pi} S \ (\delta_{ij} + A_{ij}/\rho), \ i, \ j = 1, 2 \tag{55a}$$

$$3D: \ d_{ij} = \frac{V}{4\pi} (\delta_{ij} + A_{ij}/\rho), \ i, \ j = 1, 2, 3 \tag{55b}$$

where S is the cross sectional area
V is the body volume
δ_{ij} is the Kronecker delta function $= 0$ if $i \neq j$ and $= 1$ if $i = j$
ρ is the fluid density.

In the presence of the free surface, it is the damping coefficient B_{ij} (and not the added mass coefficient A_{ij}) which gives rise to far field waves, where A_{ij} and B_{ij} are given by

$$\omega^2 A_{ij} - i\omega B_{ij} = -\rho \int_c \frac{\partial}{\partial t} (\phi_j \, e^{i\omega t}) \, n_i \, dl. \tag{56}$$

By using Green's theorem on the radiation potential ϕ_j and its complex conjugate $\bar{\phi}_j$, Newman[2] obtains the following relation between the damping coefficient and the two-dimensional far field wave amplitude A_j

$$B_{jj} = (\rho g^2/\omega^3) \ |A_j|^2, \ j = 1, 2, 3, 4. \tag{57}$$

By using far field expressions for the source and dipole, he further obtains the following relations between A_j and the resultant inner singularity strengths

$$\begin{aligned} A_j &= \frac{1}{2} (\omega/g) \ \sigma_j, \ j = 1, 3 \\ &= -\frac{1}{2} \, i(\omega k/g) \ \mu_j, \ j = 2, 4 \end{aligned} \tag{58}$$

where σ_j and μ_j are respectively the resultant inner source and dipole strengths which give rise to two-dimensional waves. By combining Eqs. (57) and (58), the desired relation between the singularity strengths and the damping coefficient is as follows

$$\sigma_j = 2\sqrt{\omega B_{jj}/\rho} \tag{59a}$$

$$\mu_j = \frac{2i}{k}\sqrt{\omega B_{jj}/\rho}. \tag{59b}$$

The above formulation applies for $j = 1, 2, 3, 4$. Equations (33) and (34) show that ϕ_5 and ϕ_6 are entirely in terms of ϕ_3 and ϕ_2, respectively. In the case of the diffraction potential ϕ_7, the "damping coefficient" may be related to the component of the diffraction force F_j^D which is 90 degrees out of phase with the incident wave. F_j^D is calculated in SMP by using the Haskind relation, Eq. (38).

5.3 Kochin Function Approach

The third approach is to characterize the singularity distribution over the entire hull by the Kochin function $H_j(k, \theta)$ which may be represented in the following two ways

$$H_j(k, \theta) = \iint_S \left(f\frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial f}{\partial n} \right) dS, \; j = 1, \ldots, 7 \tag{60}$$

$$H_j(k, \theta) = -\iint_S \gamma_j(\xi, \eta, \zeta)\, f\, dS, \; j = 1, \ldots, 7 \tag{61}$$

where θ is the direction of wave propagation with the x-axis

γ_j is the strength of the singularity at ξ, η, ζ

$$f = e^{[kz + ik(\xi\cos\theta + \eta\sin\theta)]}, \tag{62}$$

is a wave elevation function at depth z below the free surface for a wave propagating in the θ-direction. The Kochin function may be used in a variety of ways, including the evaluation of the forces acting on an oscillating body averaged over a period. Of principal interest in the present study is the fact that in the form given by Eqs. (61) and (62) the Kochin function represents an average of the singularity strengths weighted by the wave elevation function f. In the following section, an extremely simple relationship is given for the far field wave elevation in terms of the Kochin function.

6. FAR FIELD WAVE ELEVATION

6.1 Asymptotic Evaluation for Line of Singularities

In the first two approaches given above, the far field wave elevation reduces to the asymptotic evalution of the following equation

$$\zeta_j e^{i\omega t} = -\frac{1}{g}\frac{d}{dt}(\phi_j e^{i\omega t}) = -\frac{1}{g}\left(i\omega - U\frac{\partial}{\partial x} \right)\phi_j e^{i\omega t}, \; j = 1, \ldots, 7 \tag{63}$$

where ϕ_7 is composed of the antisymmetric potential ψ_2 and the symmetric potential ψ_3. Since the outer potentials are approximated by a line of oscillating sources and dipoles, Eq. (47), the problem essentially reduces to the asymptotic evaluation of Eq. (63) for a single oscillating source or dipole and then summing over all the singularities

$$\zeta_j \approx -\frac{1}{g}\left(i\omega - U\frac{\partial}{\partial x} \right) \sum_{m=1}^{M} q_{jm} \frac{\Delta L_m + \Delta L_{m+1}}{2} G_{3D}, \; j = 1, 3, 5 \tag{64a}$$

$$\zeta_j \approx -\frac{1}{g}\left(i\omega - U\frac{\partial}{\partial x} \right) \sum_{m=1}^{M} d_{jm} \frac{\Delta L_m + \Delta L_{m+1}}{2} \frac{\partial G_{3D}}{\partial y}, \; j = 2, 4, 6 \tag{64b}$$

where M is the number of cross sections or strips and ΔL_m is the axial distance between the mth and $(m-1)$st strips.

In addition to the general case $U > 0$ shown in Eqs. (63) and (64), the special case $U = 0$ is also discussed at some length since it is considerably simpler. One simplification for $U = 0$ is that $\partial/\partial t = i\omega$, resulting in the following simpler expression for ζ_j

$$\zeta_j = -\frac{i\omega}{g}\,\phi_j. \tag{65}$$

That is, the value of ζ_j is simply an algebraic factor times the value of ϕ_j. The principal reason for the simplicity of the $U = 0$ case is the considerable reduction in the complexity of the expressions for the source potentials as compared to the $U > 0$ case. Wehausen and Laitone[22] give the following formulas for the potentials G^0_{3D} and G^U_{3D} for oscillating stationary and translating three-dimensional sources placed at $x = a$, $y = b$, $z = c$

$$G^0_{3D} = \frac{1}{r} + PV\int_0^\infty \frac{k' + k}{k' - k}\, e^{k'(z-c)}\, J_0(k'R)\, dk' \tag{66}$$

$$G^U_{3D} = \frac{1}{r} - \frac{1}{r_1} + \frac{2g}{\pi}\int_0^\pi d\theta \int_0^\infty dk'\, F(\theta, k')$$

$$F(\theta, k') = \frac{k' e^{k'[(z-c)+i(x-a)\cos\theta]}\cos[k'(y-b)\sin\theta]}{gk' - (\omega + k'U\cos\theta)^2} \tag{67}$$

where only the real parts of the source potentials have been written

$r = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$

$1/r$ is a Rankine or infinite fluid source

PV denotes a principal value evaluation of the integral

$R = \sqrt{(x-a)^2 + (y-b)^2}$

J_0 is the Bessel function of order 0

$r_1 = \sqrt{(x-a)^2 + (y-b)^2 + (z+c)^2}$.

The expression for the integral term in Eq. (67) is a considerable abbreviation of the complex expression given in [22]. The actual integration interval is divided into three parts to account for the singularities arising due to the various zeroes of the denominator of the wave function F.

Asymptotic expressions for the behavior of G^0_{3D} for large values of R have been obtained by a number of authors (for example, [22], [23], [24])

$$G^0_{3D} e^{i\omega t} \sim -2\pi e^{k(z-c)} \sqrt{\frac{2k}{\pi R}}\, e^{i(Rk-\pi/4)}\, e^{i\omega t} + 0\left(\frac{1}{R}\right) \tag{68}$$

where $\sim$ denotes an asymptotic approximation. The wave elevation ζ is then obtained by using Eq. (65) and taking the real part

$$\zeta = \mathrm{Re}\,\frac{i\omega}{g}\, G^0_{3D} \sim \frac{\omega}{g}\, 2\pi\, e^{k(z-c)} \sqrt{\frac{2k}{\pi R}} \sin\left(Rk - \frac{\pi}{4}\right). \tag{69}$$

Equation (69) is simply an expression for ring waves with center at the pulsating source and decaying at the rate $\sqrt{1/R}$. It should be noted that even though the pattern is circular around a given source, the pattern around an entire ship, which has sources placed along the entire length L, will not be circular. Far field wave elevation for the dipole $\partial G^0_{3D}/\partial y$ is presumably obtained by simply taking $\partial/\partial y$ of Eq. (69).*

*Olver[25] points out that derivatives of asymptotic relations are not always permissible.

Newman[11] gives several references which investigate the far field behavior of G_{3D}^U for large values of R. These include studies by Brard,[26] Becker,[27] Eggers,[28] and Hanaoka.[29] However, these references do not derive complete asymptotic relations. Instead, they largely focus on deriving certain general features of the far field wave pattern. All of these studies as well as the work by Wehausen and Laitone,[22] show that the behavior of the wave pattern is quite different, depending on whether τ is greater or less than 1/4 where τ is given by

$$\tau = \frac{\omega U}{g}. \tag{70}$$

Thus, there is the added complexity of more than one type of asymptotic behavior.

Two approaches will be concurrently pursued in the investigation of the far field wave elevation for the general case $U > 0$. The more straightforward approach is to continue a more careful study of [26] to [29] and additional references which investigate the far field wave elevation due to G_{3D}^U. The second overall approach would be to make use of the extreme simplicity of the $U = 0$ case given by Eqs. (66), (68), and (69) or reduce the extremely complex $U > 0$ case given by Eq. (67). An obvious first step is to use Eqs. (63) and (64), which contain the effect of U in the time derivative, on the asymptotic form for G_{3D}^0. It may be argued that this is an inconsistent approach in that U is used in one part of the calculation (the evaluation of ζ) and neglected in another part (the free surface condition). This inconsistent approach has been used by Beck[30] and Lee, O'Dea, and Myers[31] (who use the two-dimensional source G_{2D}^0) to calculate the near field wave pattern next to the ship hull. Also, SMP uses the inconsistent approach of neglecting U in the free surface condition (Eq. (30)) to calculate the radiation potentials but then including U in calculating the resultant forces (Eq. (36)), which are reasonably accurate. The accuracy of this inconsistent approach for the far field wave elevation will be carefully investigated.

A second, more complex, step would be to reduce the complexity of G_{3D}^U by deriving solutions for small values of U in the free surface condition, Eq. (7). One approach would be to neglect the term involving U^2 in Eq. (7). Another approach would be to approximate the terms involving U and U^2 in Eq. (7) by known functions, such as G_{3D}^0.

6.2 Asymptotic Evaluation for Kochin Function

Finally, the calculation of the far field wave elevation using the Kochin function H (Eqs. (60) and (61)) is briefly discussed. Wehausen and Laitone[22] give the following asymptotic evaluation of ζ_j derived by Kochin, which is restricted to the $U = 0$ case

$$\zeta_j(R, \theta) \sim \frac{\omega}{g} \sqrt{\frac{k}{2\pi R}}\, \overline{H}_j(k, \theta) \sin\left[Rk - \frac{\pi}{4}\right], \quad j = 1, \ldots, 7 \tag{71}$$

where $\overline{H}_j$ is the complex conjugate of H_j. Equation (71) shows that ζ_j is very similar in form to the wave elevation ζ for a source of unit strength, given in Eq. (69). If $z = c = 0$ in Eq. (69), which means that ζ is evaluated at the free surface for a source placed at the free surface, the following expression for the ratio ζ_j/ζ results

$$\frac{\zeta_j}{\zeta} = \frac{1}{2\pi} \sqrt{\frac{1}{2}}\, \overline{H}_j(k, \theta). \tag{72}$$

The right hand side of the above equation may be thought of as the "strength" of a single resultant "source-like" singularity which is equivalent to the weighted average of all the sources on the ship hull. Equation (71) then gives the wave elevation due to this weighted singularity. In the first two approaches described above, the contribution of each source to the far field wave is first evaluated and the resultant wave elevation is then obtained as the sum of all the individual contributions. The discussion following Eq. (69), which states that the resultant wave pattern for the entire ship is not circular, is confirmed by the fact that $\overline{H}_j$ is a function of the direction θ.

7. SUMMARY

A somewhat detailed description is given of the linearized formulation for the radiation and diffraction problems. It is pointed out that even the linearized formulation is quite complex. Various simplified approaches including thin ship, flat ship, and long wavelength theories are shown to be either unsuccessful or inapplicable to the present interest of calculating the far field wave pattern.

The high frequency (low wavelength) slender body strip theory, as implemented in the widely used DTNSRDC Ship Motion Program (SMP), is identified as a simple yet relatively accurate approach for calculating the flow field near the ship hull. This theory neglects the forward speed effect in the free surface condition as well as interaction in the longitudinal direction. A unified strip theory which accounts for longitudinal interaction is mentioned as a possible later approach. Strip theory uses two-dimensional sources to solve the radiation problem due to ship oscillations in the vertical plane (surge, heave, and pitch) while dipoles are used for oscillations in the lateral plane (sway, roll, and yaw).

It is pointed out that the diffraction problem generally leads to the solution of a Helmholtz equation instead of Laplace's equation for the radiation problems. However, for wavelengths which are intermediate between ship beam (or draft) and ship length, Laplace's equation may again be used. In this case, the solution for the diffraction problem reduces to the solution of a pair of potentials which are similar to the radiation potentials for sway and heave.

It is shown that the near field two-dimensional singularities are not appropriate for calculating the far field wave elevation. Two somewhat similar approaches are indicated for converting the near field singularity strengths to a line of three-dimensional far field singularity strengths. In one approach, it is shown that the strength of the far field singularity at a given longitudinal station may be approximated by the resultant inner singularity strength for the cross section at the given station. This approximation is accurate to high order for the lateral modes and is reasonably correct for the vertical modes for long wavelengths of the order of ship length and short wavelengths of the order of ship beam or draft. A more complex integral equation approach may be used if more accuracy is desired for the vertical modes. In the second approach, it is shown that the far field singularity strength may be obtained from the calculated (or measured) damping force acting on the cross section.

Once the far field singularity strengths have been determined, the far field wave elevation may be obtained by first calculating the contribution due to each singularity and then summing over all the singularities. It is shown that the potential for the stationary pulsating source is much simpler than the corresponding potential for the pulsating source moving at forward speed U. In particular, extremely simple expressions have been derived for the asymptotic far field behavior of the potential and wave elevation for the stationary pulsating source. Corresponding expressions for the dipole may be conveniently obtained by taking lateral derivatives $\partial/\partial y$ of the results for the source. Asymptotic expressions do not appear to have been derived for the pulsating source with $U > 0$. Instead, most studies in this area (which are often written in a foreign language) attempt only to identify major features of the complex wave pattern. It is pointed out that the most fruitful approaches in this case probably lie in simplifying the potential for $U > 0$ for, say, small values of U to capture the essential effect of U in the free surface condition. A particularly simple first step would be to evaluate wave elevation by including U in the time derivative operation on the potential for the stationary source.

An alternate way of obtaining the far field wave pattern is to first calculate the Kochin function which is a weighted average of the singularity strengths over the ship hull. For the case of a stationary ship, $U = 0$, the expression for the resultant far field wave pattern is similar to the expression for a single source with "strength" which is proportional to the complex conjugate of the Kochin function.

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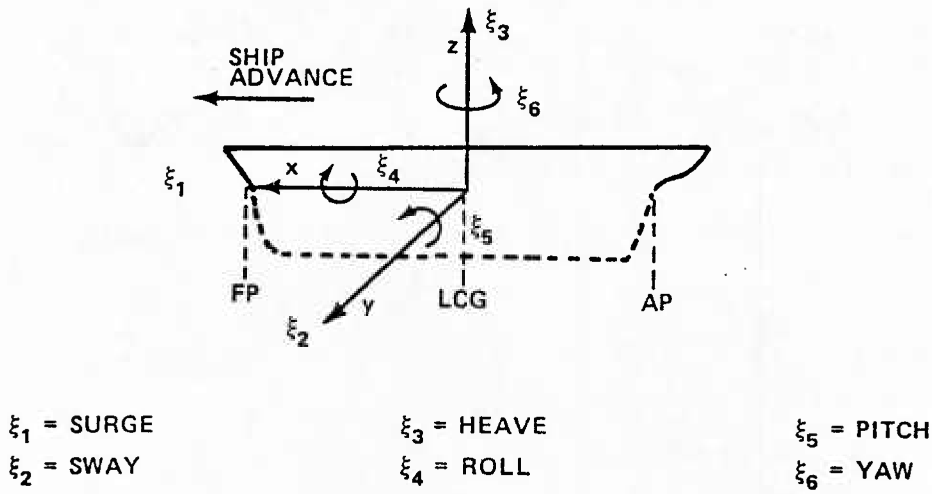

Fig. 1 — Definition of coordinate system and six modes of ship motion

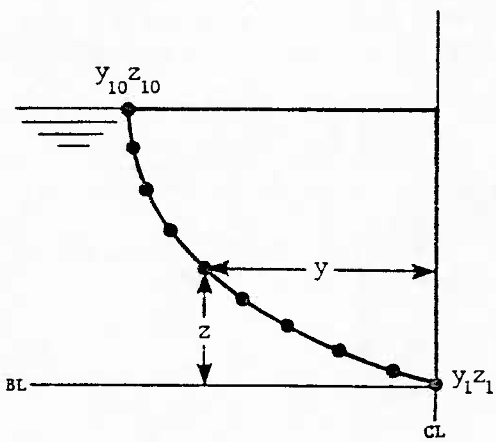

Fig. 2 — Typical station offset distribution